Fixed Income Derivatives
Final Exam Solution - 2011

Introduction

This exam solution is accompanied by a spreadsheet that contains all the relevant tables and graphs. This also shows at least one way of implementing the required models. Please refer to the spreadsheet for the numerical results.

From the course description:

> *As the course is oriented towards the use of derivatives in practice, students are required to demonstrate a thorough understanding of all aspects surrounding derivatives - from the legal framework to the practical implementation of basic numerical routines using VBA/Excel - to obtain the grade 12.*

For this specific exam, this means that students are required to demonstrate the ability to implement, calibrate and apply the models from the course. Furthermore, successful students are expected to demonstrate some intuition on the pricing and risk management calculation that they perform.

1 The Swap Curve

1.1 Estimating the swap curve

The calibrated zero coupon curves and the requested graphs can be seen found below and in the solution spreadsheet as well. The calibrated rates should be within ± 0.5 bps.

Table 1: Calibrated zero coupon rates.

Knot Points	EUR Forward	EUR Discounting
02-Apr-12	1.8771%	1.6217%
01-Apr-13	2.3569%	2.1068%
01-Apr-14	2.6831%	2.4621%
01-Apr-16	3.1021%	2.9285%
02-Apr-18	3.3856%	3.2468%
01-Apr-21	3.6714%	3.5950%
03-Apr-23	3.8357%	3.7861%
01-Apr-26	4.0078%	3.9949%
01-Apr-31	4.0820%	4.1440%
01-Apr-41	3.7886%	3.9491%

Figure 1: Yield curves in 3M points.

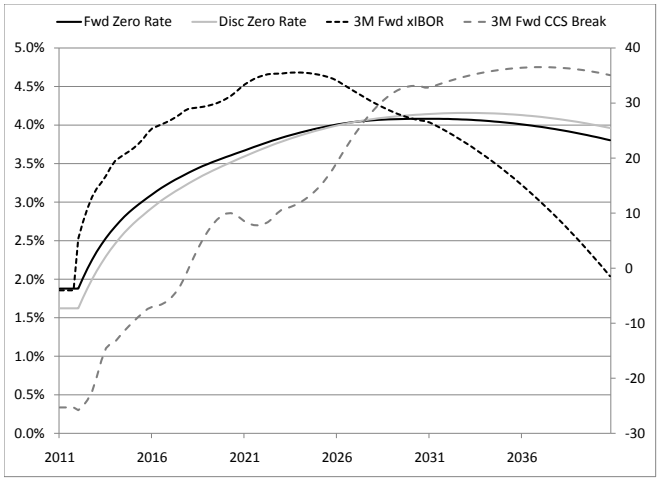

Table 2: Portfolio A

Notional	Start	End	Pay/Rec	Fixed Rate	PV (EUR)
125,000,000	01-Jan-14	01-Jan-19	Payer	2.602%	7,374,423
78,000,000	01-May-12	01-May-14	Payer	3.087%	132,124
500,000,000	01-Apr-15	01-Apr-25	Receiver	4.013%	-15,281,296
150,000,000	01-Nov-11	03-Nov-31	Payer	4.228%	-1,861,904
100,000,000	01-Apr-16	01-Apr-36	Receiver	4.101%	-3,149,462
65,000,000	01-Aug-12	01-Jan-16	Receiver	3.670%	343,575
				Total	-12,442,540

Table 3: Portfolio B

Notional	Start	End	Pay/Rec	Fixed Rate	PV (EUR)
58,000,000	01-Aug-11	01-Aug-16	Receiver	2.783%	-1,299,737
250,000,000	01-Nov-12	03-Nov-14	Payer	3.208%	923,954
90,000,000	01-May-13	01-May-23	Payer	3.551%	4,431,487
200,000,000	03-Sep-12	03-Sep-42	Receiver	3.740%	-10,053,636
50,000,000	01-Mar-12	01-Mar-27	Receiver	3.826%	-1,868,938
115,000,000	01-Dec-11	01-Dec-21	Payer	4.027%	-1,525,242
				Total	-9,392,111

1.2 Marking swap portfolios to market

The value of each swap and the total portfolio mark-to-markets can be in table 2 and table 3. For the two portfolios, a margin of error of up to EUR 200,000 is accepted (which corresponds roughly to each portfolio's Dv01).

1.3 Reporting risk on swap portfolios

The zero rate delta vectors are shown below. Correct answers have a net DV01 of EUR $\pm 1,000$. Looking at the market rate delta vectors, we an see that both portfolios are —

on a net basis — positioned for lower interest rates by roughly EUR 200,000 per bps. For portfolio A this exposure is mainly coming from the 12Y and 15Y segment while portfolio B is mostly exposed to the 30Y segment. Portfolio A has however also got an implicit curve position. In particular, the portfolio is positioned for a flatter curve between 3-5Y and 12-15Y. In terms of the CCS risk in the two portfolios, portfolio B is positioned for an increase the EUR/USD CCS spread. Although this exposure is spread across the curve, most of it can be attributed to the 20-30Y segment. For portfolio A, the CCS exposure leaves the portfolio positioned for a curvature in the CCS spread curve (shorter- and longer end spread to increase, while the middle segment should decrease).

Students should demonstrate the ability to interpret the risk numbers. Full credit should be given to students who correctly identify the portfolio exposure. Even if students did not calibrate their initial curves with precision, the risk numbers — due to the limited convexity in these swap portfolios — should be identical to the solution if the correct method has been applied.

Table 4: Zero rate delta vector, portfolio A

Maturity	Fwd	Disc	Net
1Y	-10,418	-34	-10,452
2Y	-34,434	295	-34,138
3Y	120,911	721	121,633
5Y	100,066	120	100,186
7Y	35,716	166	35,882
10Y	9,830	4,208	14,038
12Y	-204,151	6,153	-197,998
15Y	-268,723	-522	-269,245
20Y	52,526	-643	51,883
30Y	-29,556	-631	-30,187
Total	-228,233	9,834	-218,399

Table 5: Zero rate delta vector, portfolio B

Maturity	Fwd	Disc	Net
1Y	752	106	858
2Y	-46,832	-416	-47,248
3Y	82,497	524	83,021
5Y	-6,010	639	-5,371
7Y	-12,022	420	-11,601
10Y	56,872	1,046	57,918
12Y	84,353	4,721	89,074
15Y	-70,233	8,534	-61,700
20Y	-81,041	-1,657	-82,697
30Y	-214,444	-8,613	-223,057
Total	-206,107	5,304	-200,803

Table 6: Market rate delta vectors

Instrument	Maturity	Portfolio A	Portfolio B
IRS	1Y	-10,456	685
	2Y	-34,197	-47,177
	3Y	123,967	84,112
	5Y	112,492	-3,734
	7Y	65,828	-9,416
	10Y	54,482	73,984
	12Y	-213,094	130,123
	15Y	-357,537	-60,290
	20Y	88,703	89
	30Y	-48,338	-358,949
CCS	1Y	116	57
	2Y	1,363	111
	3Y	1,928	377
	5Y	1,026	663
	7Y	-2,345	500
	10Y	-1,264	380
	12Y	1,445	133
	15Y	-1,183	326
	20Y	-647	3,213
	30Y	1,360	3,401

1.4 Quantifying portfolio risk

The question is solved by multiplying the set of 1-day changes in market rates to the market rate delta vectors found above. From this we can estimate a distribution of 1-day profits and losses from which we calculate the VaR and ES as percentiles, respectively, conditional means. From the table below, we can see that in a VaR risk sense, portfolio A is more risky compared to portfolio B. This intuitively means that on "average" days portfolio A presents a larger risk. On "extreme" (in the expected shortfall sense) days, the two portfolios seem to have almost the same risk with portfolio A again being slightly more risky.

Table 7: Portfolio risk

	Value-at-Risk	Expected Shortfall
Portfolio A	-1,922,309	-2,725,906
Portfolio B	-1,637,379	-2,707,653

Results in this sub question will obviously depend on the market rate delta vector calculated above. Full credits should be given to students who match the risk figures to EUR $\pm 100,000$.

2 Pricing CMS products

2.1 Calibrating SABR models

Students should set up a calibration over the implied volatilities given in the data. Minor numerical deviations can arise from this calibration, so the following margin of error should be accepted: $\sigma_0 = \pm 0.25\%$, $\epsilon = \pm 5.00\%$ and $\epsilon = \pm 5.00\%$. The calibrated parameters can be found in the table 8.

Table 8: SABR calibration

Swaption	α	σ_0	ϵ	ρ
1Y30Y	52.00%	4.85%	66.08%	-50.48%
2Y30Y	52.00%	4.75%	62.53%	-40.54%
5Y30Y	52.00%	4.32%	53.83%	-32.51%

2.2 Pricing a CMS swap

In this question, students should set-up the schedule for the CMS swap and use a combination of the VBA functions to price the product. It is worth noting, that it is not necesarry to know a specific notional to price the CMS swap at par (i.e. with a PV of zero) — any bullet notional goes for this purpose. The par spread in the CMS swap is 137.4 bps. Here, a margin of error of up to ± 3 bps should be accepted as the pricing will depend on the SABR calibration.

Table 9: CMS pricing

Item (unit notional)	PV
3M EURIBOR Leg	-0.159750
CMS Leg	0.224457
PV of additional bp	0.000471
CMS Par Spread (bps)	137.4

2.3 Static CMS replication

The VBA coding in this question is very easy since it is just a question of exchanging a parameter that is hard coded into the original function. The coding effort needed here is thus minimal. An example can be seen in the solution spreadsheet. Students should be able to replicate the below numbers within the same margin of error as above (i.e. ± 2 bps). Importantly, the spreads should be monotonically increasing in the upper bounds.

The CMS spread depends on the integration bound via the static replication via (high strike) payer swaptions. This replication is required due to the convexity difference the 30Y swap and the 3M CMS swaplet. When hedging the — close to linear — CMS swaplet with the much more convex 30Y swap, a convexity gap arises. The size of this gap in increasing with level of interest rates. To close this gap, a static hedge can be established using swaptions. Theoretically, the static hedge will require swaptions with infinitely high strikes, however the value of these becomes negligible beyond some cutoff point. This is illustrated by the fact that the CMS spread is increasing in the upper bound but is clearly also converging to the "true" par spread.

Table 10: CMS replication

Upper bound for integration	CMS par spread
5%	124.7
10%	134.8
15%	136.5
20%	137.4
30%	138.2

3 The capped floater

3.1 The cap in the capped floater bond

Since the plain vanilla cap has a pay-off of the type $\max(\text{xIBOR}-K, 0)$ (rahter than $\max([\text{xIBOR}+a] - K, 0)$) we calculate the plain vanilla strike as $4.40\% = 5\% - 60$ bps. The mid mid-market value of this cap on the given curves and SABR model is DKK 4,195,623. A margin of error of DKK $\pm 50,000$ should be accepted.

3.2 The risk profile of the capped floater

Table 11: Capped floater net zero rate delta vector

1Y	155
2Y	2,594
3Y	5,556
5Y	5,349
7Y	1,481
10Y	-51,720
12Y	-390
15Y	0
20Y	0
30Y	0
Total	-36,974

3.3 Capped floater asset swap spread

In this question, the margin accpetable margin of error is ± 1 bps on the ASW spread.

Table 12: 5% Capped floater ASW calculation

Dirty (=Clean) price	97.50
Value of Capped floater coupons	33,331,472
Value of price difference to par	2,500,000.00
Value of 6M Floater flat	-32,387,779
Net	3,443,692
Value 1 bp on floater	85,655.25
ASW	40.20

Figure 2: Capped floater net zero rate delta vector.

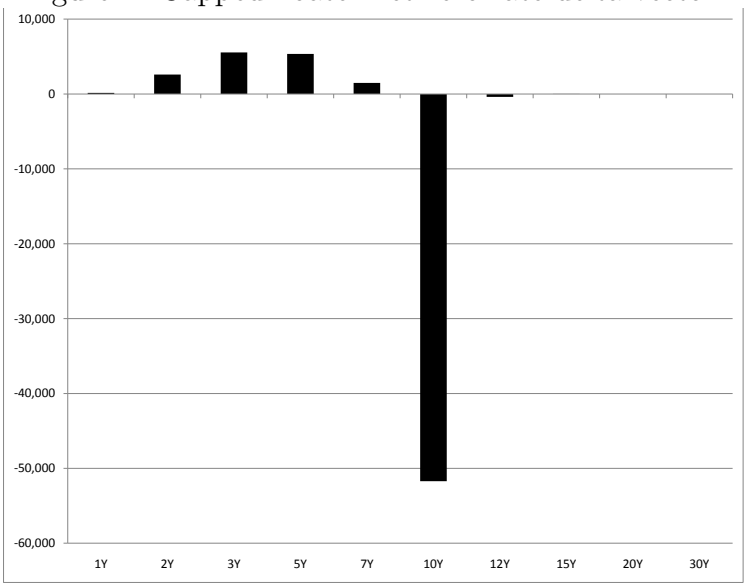

Table 13: 6% Capped floater ASW calculation

Price	99.5
Value of 6M CIBOR (flat)	32,387,779
Value of borrower spread (60 bps)	5,137,438
Value of Cap	-2,194,096
Value of Capped floater coupons	35,331,121
Value of price difference to par	500,000
Value of 6M Floater flat	-32,387,779
Net	3,443,341
Value 1 bp on floater	85,655.25
ASW	40.20

3.4 Capped floater product development

The borrower spread resulting in a par-par ASW of 40.20 bps when the capped floater is trading at 99.50 under the given market conditions is 60.0 bps. The borrower spread is thus exactly the same as for the strike 5% capped floater. Why is that the case?

The answer to this question is two-fold. First, the borrower spread is effectively what is financing the purchase of the cap. Since a cap with a higher strike should be less expensive (the borrower is buying less protection against rising interest rates), the borrower spread should ceteris paribus be smaller for the 6% capped floater compared to the 5% capped floater. However, the price of the bond also plays a role — why everything else is not equal in this case. Since the price of the strike 6% bond is higher, the revenue generated by selling the bond is higher. In order to finance a fixed amount, you would need to sell fewer bonds at the price of 99.50 (the 6% strike bond) relative to a price of 97.50 (the 5% strike bond). Ceteris paribus, a higher bond price thus implies a lower borrower spread to keep the effective interest cash flow paid by the borrower unchaged. As it turns out, these two effects exactly cancels out for the two bonds in question and the borrower spread is therefore the same.

3.5 A callable capped floater

As demonstrated by the zero delta vector, the price of the capped floater is increasing (locally at least) when rates fall. The same holds true globally because of the borrower spread. Recalling that a floating rate note that is discounted at the same interest rate as it pays has par value on the fixing date, we can conclude that it is a wedge between the coupon rates and the discounting rates than can drive the price of the (uncapped) floater above par. This wedge can arise either from the borrower spread or from lower discounting rates (resulting in a lower (perhaps negative) ASW spread).

Intuitively, the value of the borrower spread will at zero interest rates give rise to value of approximately DKK 6 per every DKK 100 of notional (60 bps being paid undiscounted for 10 years). So any combination of low ASW spreads or low rates in general will push the price of the capped floater higher.

Finally, as the bond owner is short the embedded cap, lower implied volatility will also help drive the price of the capped floater higher. It is however worth noting that as rates drop (moving the forward CIBOR rates further away from the cap strike), the vega sensitivity of the bond will diminish.

Since the question is posed rather broad, the answer is somewhat open. Students should recognize the points above although any well guided intuition should be rewarded.